The Effect of Diameter on the End Correction of an Open-Ended Tube

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IB Physics HL 2

Introduction

When a sound of a specific frequency is emitted from a frequency emitter and positioned at the end of an open-ended tube, a standing wave is created as a result of interference. Due to the nature of the tube, the positions of maximum displacement (anti-node) are situated at the open ends, while the position of zero displacement (node) is at the center of the tube. From this, the relationship between the length from the anti-node at one end to the anti-node at the opposite end and the wavelength of sound at the fundamental harmonic was found, it is as follows:

$$L = \frac{\lambda}{2}, \qquad (1)$$

where L is the length of the tube in meters and λ is the wavelength of the emitted wave.

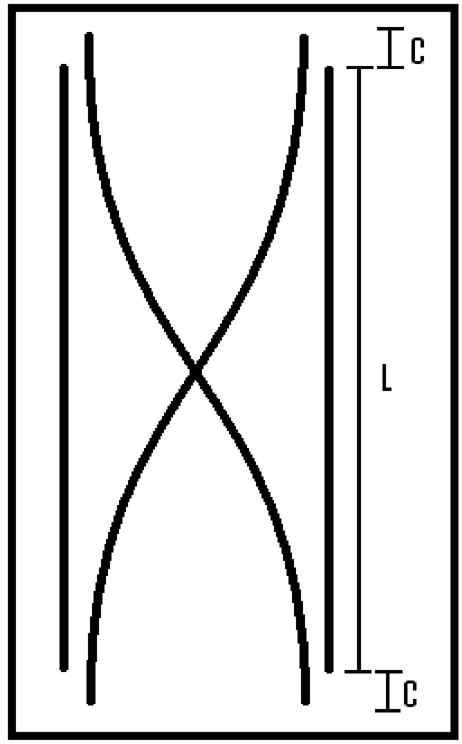

Figure 1: The end correction of an open-ended tube.

However, due to the reflective properties of waves at the open ends of a tube, the anti-nodes extend beyond the open ends[1]; this phenomenon is known as the end correction (Figure 1). Thus, by modifying equation 1 to include the end correction, it is as follows:

$$L + 2C = \frac{\lambda}{2}, \qquad (2)$$

where L is the length of the tube in meters, C is the end correction of the tube in meters, and λ is the wavelength of the emitted wave.

Another equation was found relating the wavelength of a wave, to the frequency of the wave, where the relationship is non-causal. The equation is known as the wave relationship:

$$v = f\lambda, \qquad (3)$$

Where v is the speed of sound in air, f is the frequency of the wave, and λ is the wavelength of the emitted wave. By manipulating equation 3 to show end correction in terms of the length, and the frequency it turns out as follows:

$$C = \left(\frac{v}{2f} - L\right)/2, \qquad (4)$$

Therefore, from the equation above, the end correction is expected to be dependent on the frequency and the length of the tube. However, it was found that the end correction of an un-flanged open-ended tube is dependent upon the radius of the tube, where the end correction can be calculated to approximately 0.3 times the diameter of the tube[1]. The source omits the relationship (if any) between the end correction of the tube, the frequency of the emitted sound, and the length of the tube. Therefore, according to the theory above, it is expected that the end correction will increase linearly with an increase in diameter of a tube.

Design

Research Question

How does the diameter of an open-ended un-flanged tube affect its end correction?

Variables

The independent variable was the diameter of the open-ended un-flanged tube. The dependent variable was the end correction of the tube. Several controls were kept constant during the research to ensure the highest accuracy possible. Tubes with equal lengths of 39.7 (±0.1 cm) were used throughout the experiment and the resonance was always measured at the first harmonic to make sure that the length of the standing wave was kept constant The method of measuring the resonance was kept the same throughout the entire research by keeping the tube perpendicular to the speaker so that the position of the speaker would not interfere with the nature of the wave. The temperature was kept constant at 26 (±0.1 °C) by keeping the air conditioners turned on. This was to ensure that the speed of sound would remain constant throughout the entire research. Lastly, the same frequency emitter was used to ensure constant systematic errors in precision.

Materials and Procedure

First, 6 open-ended P.V.C. pipes with different diameters were obtained and cut with a hacksaw to equal lengths of 39.7 (±0.1 cm). A caliper was used to measure the inner diameter of the tube. Next, a frequency emitter that emitted a frequency in the form of a sine-wave was attached to a speaker and placed at the tip of the pipe, at a perpendicular angle. The frequency was gradually changed until resonance occurred at the highest possible frequency. This was to ensure that the fundamental harmonic was always being observed.

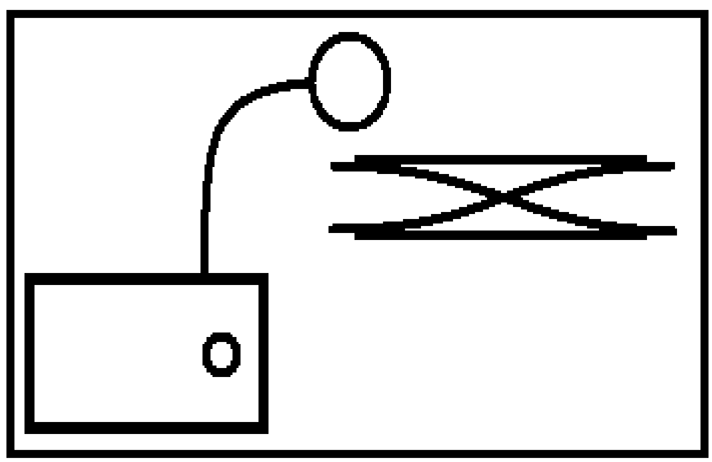

Figure 2: The experimental set-up.

This was repeated for six different diameters ranging from 14 mm to 56 mm; three trials were conducted for each radius to increase the precision of the research. Throughout the entire experiment, and the set-up was maintained.

Data Collection and Processing

The Diameter of the Open-Ended Tube and the Frequency of Sound at Maximum Resonance

Diameter of Tube (±0.1 mm)	Frequency of Sound at Maximum Resonance (±1 Hz)		
	Trial 1	Trial 2	Trial 3
14.4	424	424	424
18.6	418	419	420
22.7	415	416	416
29.4	410	410	409
36.5	406	407	406
55.8	399	400	399

Table 1: The diameter of the open-ended tube, and the frequency of the sound at maximum resonance. The uncertainty of the diameter and the frequency is the instrumental uncertainty of the ruler and the frequency emitter respectively.

The Diameter of the Open-Ended Tube and the Mean Frequency of Sound at Maximum Resonance

Diameter of Tube (±0.1 mm)	Average Frequency of Sound at Maximum Resonance (±1 Hz)
14.4	424
18.6	419
22.7	416
29.4	410
36.5	406
55.8	399

Table 2: The diameter of the open-ended tube, and the mean frequency of sound at maximum resonance. The uncertainty of the average distance was taken from half the largest range of all trials.

The Frequency and the Calculated End Correction (First Harmonic)

Diameter of Tube (±0.1 mm)	Calculated End Correction (*C*) (±0.1cm)
14.4	0.6
18.6	0.8
22.7	1.0
29.4	1.3
36.5	1.5
55.8	1.9

Table 3: The frequency and the calculated end correction, calculated by using equation 4.

Note: The speed of sound in air at 26 (±0.1 °C) was used: 346.87 (±0.06 m/s)[2].

Sample Calculations

i. Mean Frequency at Maximum Resonance (18.6 mm)

= (Trial 1 + Trial 2 + Trial 3) / 3
= (418 + 419 + 420) / 3
= 419 Hz

ii. Uncertainty of Mean Frequency at Maximum Resonance (18.6 mm)

= Range / 2
= (420 – 418) / 2
= 0.1 Hz
≈ 419 (±1 Hz)

iii. Calculated End Correction (18.6 mm)

$$= \left(\frac{v}{2f} - L\right) / 2$$

$$= \left[\frac{346.867}{(2\times 419)} - 0.397\right] / 2$$

= 0.008462172 m
= 0.8462172 cm

iv. Uncertainty of Speed of Sound in Air

= (Maximum Speed of Sound – Minimum Speed of Sound) / 2
= (346.925 – 346.809) / 2
= 0.058 m/s
≈ 346.87 (±0.06 m/s)

v. Uncertainty of End Correction (18.6 mm)

= *(Maximum End Correction – Minimum End Correction / 2*

$$= \left[\left(\frac{346.93}{(2\times 418)} - 0.396\right) - \left(\frac{346.81}{(2\times 420)} - 0.398\right)\right] / 2$$

= 0.0010297477 m
≈ (±0.1 cm)

vi. Uncertainty of $\mathbf{Log_{10}}$(Diameter) (18.6 mm)

= (Maximum Log_{10}(Diameter) – Minimum Log_{10}(Diameter)) / 2

$$= \left(\log_{10} 18.7 - \log_{10} 18.5\right) / 2$$

= 0.0023349391
≈ (±0.002)

vii. Uncertainty of $\mathbf{Log_{10}}$(End Correction) (18.6 mm)

= (Maximum Log_{10}(End Correction) – Minimum log_{10}(End Correction)) / 2

$$= \left(\log_{10} 0.9 - \log_{10} 0.7\right) / 2$$

= 0.0545722347
≈ (±0.05)

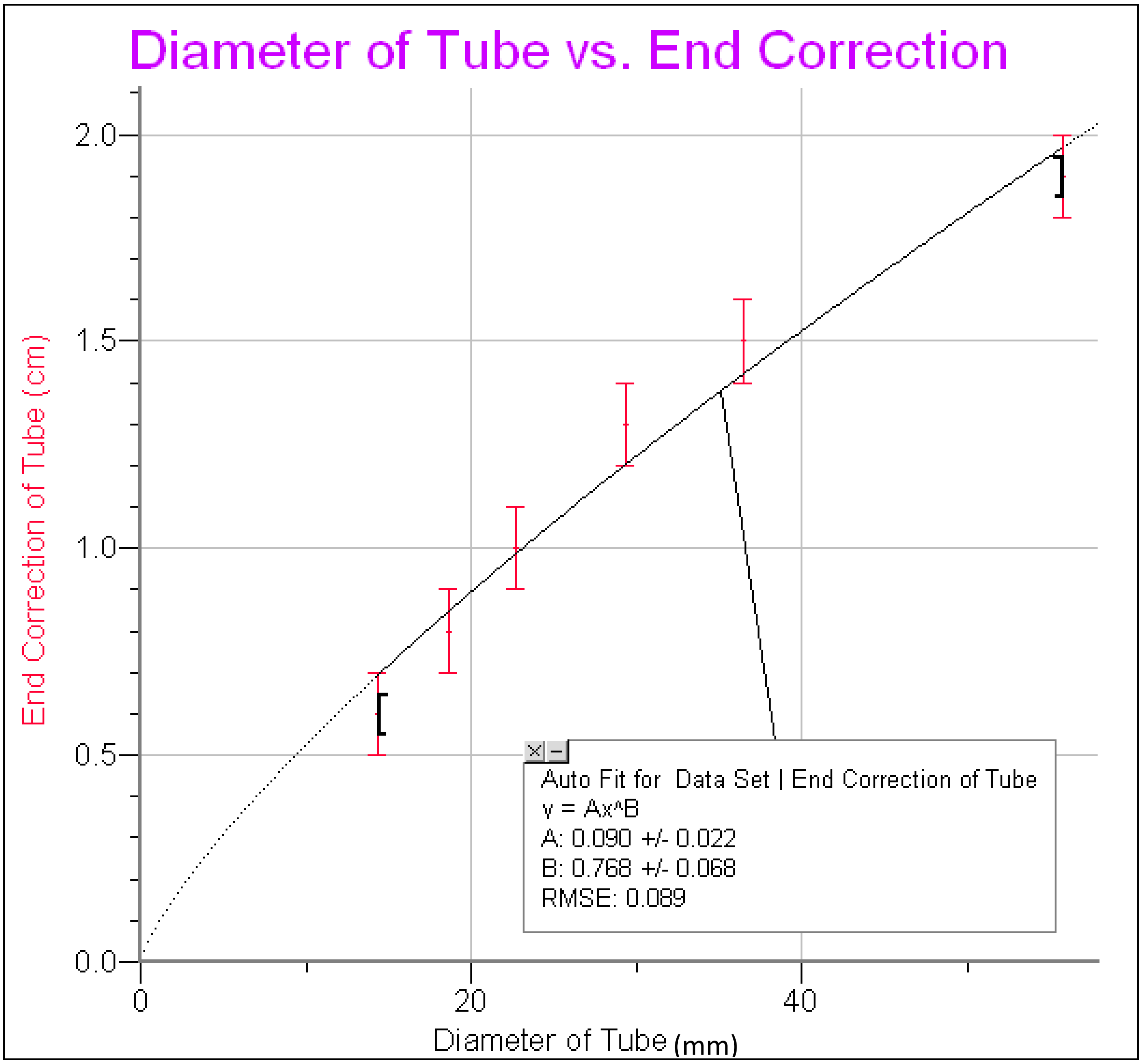

Figure 3: The diameter of the tube versus the end correction of the tube with a power fit.

A $\log_{10}$(Diameter) versus $\log_{10}$(End Correction) was graphed because it was predicted that they would have a linear relationship. The derivation of the linear equation is shown below.

$$\text{End Correction} = A \times \text{Diameter}^n$$

$$\log_{10} C = \log_{10}(A \times D^n)$$

$$\log_{10} C = \log_{10} A + \log_{10} D^n$$

$$\log_{10} C = n \log_{10} D + \log_{10} A$$

$$y = nx + b$$

Therefore the slope of a log(Diameter) vs. log(End Correction) graph signifies the power n, while the y-intercept signifies the $\log_{10}$ value of the constant A.

The Log$_{10}$(Diameter) and the Log$_{10}$(End Correction) (First Harmonic)

Log$_{10}$(Diameter) (±0.002)	Log$_{10}$(End Correction) (±0.05)
1.158	-0.22
1.270	-0.10
1.356	0
1.468	0.11
1.562	0.18
1.747	0.28

Table 4: The log$_{10}$(Diameter) and the log$_{10}$(End Correction).

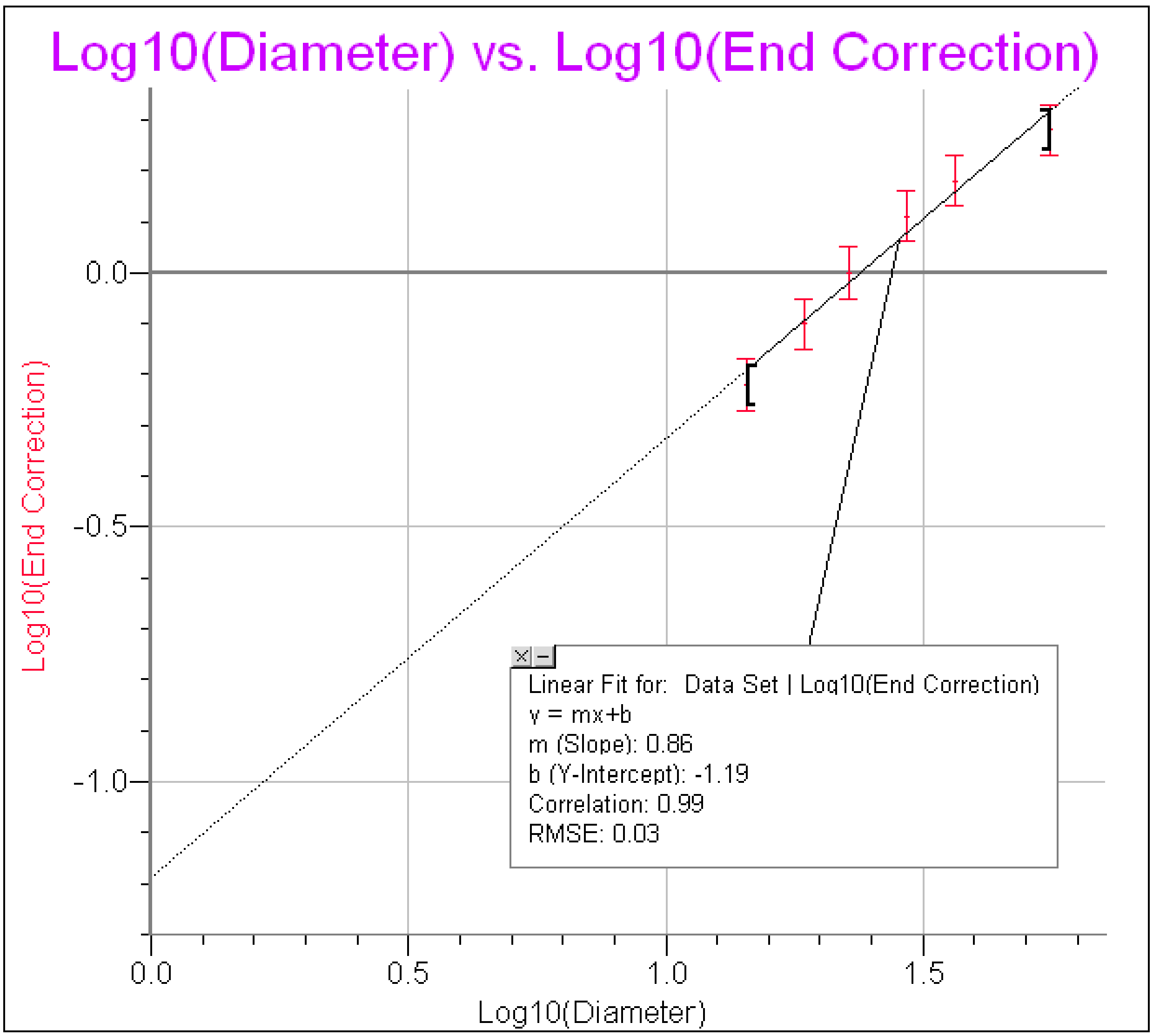

Figure 4: The log$_{10}$(Diameter) versus the log$_{10}$(End Correction) with a linear fit.

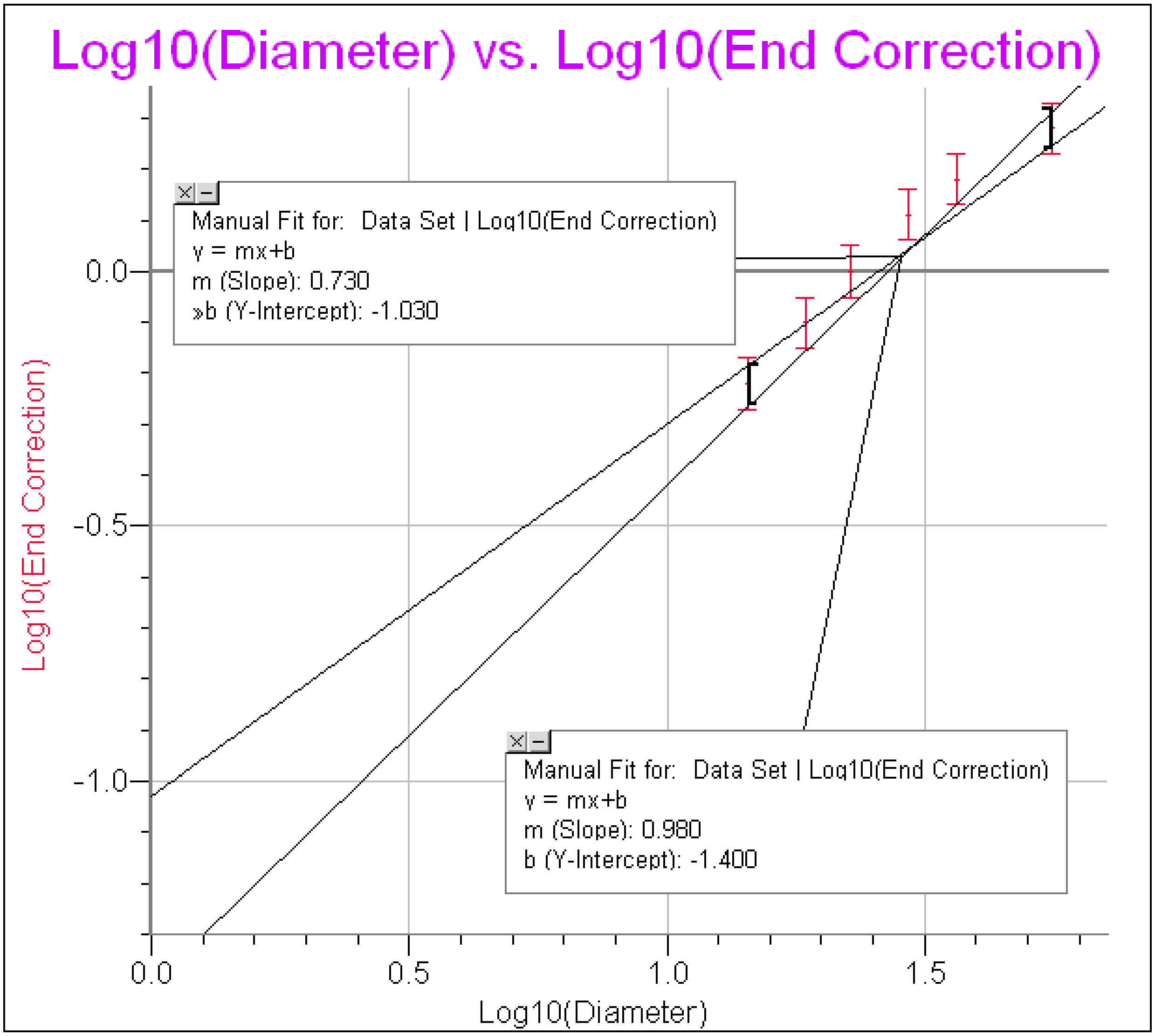

Figure 5: The $\log_{10}$(Diameter) versus the $\log_{10}$(End Correction) with 2 extreme linear fits.

Sample Calculations (Cont.)

viii. The Uncertainty of the Slope of $\mathbf{Log_{10}}$(Diameter) versus $\mathbf{Log_{10}}$(End Correction)

= *(Maximum Slope – Minimum Slope) / 2*

= (0.98 – 0.73) / 2

= 0.125

≈ (±0.1)

ix. The Uncertainty of Constant A

$$= \left(10^{Maximum\ Y-Intercept} - 10^{Minimum Y-Intercept}\right)/2$$

$$= \left(10^{-1.03} - 10^{-1.40}\right)/2$$

= 0.0267

≈ (±0.02)

Conclusion

According to figure 4 and 5, an equation of the end correction in terms of the diameter of a tube was found. The slope of each graph signifies the power *n*, while the y-intercept signifies $\log_{10}A$. The equation turned out to be

$$C = A \times D^n$$
$$C = 0.06(\pm 0.02) \times D^{0.9(\pm 0.1)}$$

The range of the 2 extreme lines in figure 5 was used to find the uncertainties of the power *n*, and the constant *A*.

Therefore the expectation that the end correction would increase linearly with the increase in diameter of the tube is not supported because there seems to be a power relationship between the end correction and the radius of the tube.

Furthermore, according to the theory that end correction is 0.3 times the diameter, the calculated end correction and the observed end correction calculated from the data of this research differed by at least 30% in all trials.

In figure 3, the x-intercept and the y-intercept approach zero, this makes sense because when the length of the tube is zero, it cannot have an end correction; a standing wave cannot be formed in a tube with length zero.

The validity of this research is limited to open-ended P.V.C. tubes lengths of 39.7 cm and diameters ranging from 14 mm to 56 mm. While it is expected that the relationship for similar situations would be the same, further investigation needs to be done to confirm this.

Evaluation

The method of measuring the position of resonance at the first harmonic was a random error that could have affected the results. The position of resonance was based on our hearing, and where we thought the resonance was loudest. This created uncertainties in the measurements and decreased the precision of this research. A sound level meter could have been used to analyze the position where the resonance was actually at its loudest.

Due to procedural uncertainties, the edges of the pipe were not cut equally, resulting in one side of the pipe being longer than the other side, therefore this was a random source of error. This could have led to uncertainties in the length of the standing wave and the reflective property of the waves, which in turn could have affected the end correction of each pipe. A machine could have been used to reduce the uncertainty in cutting, so that the edges of the pipe were uniform in length.

References

[1] http://www.britannica.com/EBchecked/topic/186703/end-correction
[2] http://www.sengpielaudio.com/calculator-speedsound.htm